METHOD OF DESIGNING A VORTICAL DIFFUSER

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A method is proposed for theoretically determining the characteristics of a vortical diffuser. Comparison of the theoretical and experimental results shows that they agree well.

Most vortical refrigerating and vacuum devices have divergent diffusers which actively affect their characteristics. The self-evacuating vortex tube (SVT) and turbine vacuum pump (TVP) could not operate without a diffuser (Figs. 1 and 2).

The hypothesis of vortex interaction [1] makes it possible to calculate the characteristics of vortex flow tubes. Here their energy characteristics are dependent on the parameter $\pi_1 = P_1/P_{ax}$.

In vortex flow tubes, the cold gaseous component is discharged into the atmosphere, so that the total expansion of the gas in the vortex π^* can be found. This is because the pressure on the axis P_{ax} is equal to the atmospheric pressure P_n.

In the case of the operation of SVT's and TVP's, this parameter is not determined, since the pressure on the axis P_{ax} is unknown. As experimental studies [2, 3] have shown, the value of π^* can be regulated within the range $\pi^* = 1$-40 by changing the geometric characteristics of the diffuser (by changing the width of the diffuser slit, shaping the latter, etc.). Thus, the total gas expansion in a vortex for vortical devices of a given type has been determined only experimentally up to the present time.

The proposed method of designing vortex units is based on an iteration method, developed by the author [4], for determining the inlet parameters of the diffuser of a vortex unit and relations describing the distribution of thermodynamic parameters in the vortex chamber [1].

Determination of the parameters at the diffuser inlet involves the use of a technique for designing a two-dimensional divergent diffuser with allowance for compressibility, viscosity, and separation losses [5]. For a divergent diffuser with a through section of a linearly varying width, we will write the design equations as follows:

$$\frac{d\lambda_r}{d\bar{r}} = \left\{-2\lambda_\varphi\left(\frac{\lambda_\varphi}{\bar{r}} + 0{,}046\chi\beta\bar{r}\varepsilon\,(\lambda_r^2+\lambda_\varphi^2)^{0.5}\lambda_\varphi\,\mathrm{Re}^{-0.2}(\varepsilon_0\lambda_{r0})^{-1}\right. - \frac{(k+1)(\lambda_{r0}\varepsilon_0)^{k-1}\,[\lambda_r(1-\beta\,\mathrm{tg}\,\gamma(\bar{r}-1)) - \bar{r}\lambda_r\beta\,\mathrm{tg}\,\gamma]}{[\lambda_r\bar{r}(1-\mathrm{tg}\,\gamma(\bar{r}-1)\,\beta]^k}\right\} \Big/ \left[\frac{\bar{r}(1-\mathrm{tg}\,\gamma(\bar{r}-1))\,(k+1)(\lambda_{r0}\varepsilon_0)^{k-1}}{(\lambda_r\bar{r}(1-\beta\,\mathrm{tg}\,\gamma(1-\bar{r})))^k} - 2\lambda_r\right], \tag{1}$$

$$\lambda_\varphi = \left\{\left[1-\left(\frac{\varepsilon_0\lambda_{r0}}{\lambda_r\bar{r}\bar{\Delta}}\right)^{k-1}\right]\frac{k+1}{k-1} - \lambda_r^2\right\}^{0.5},$$

$$\bar{\Delta} = 1-\mathrm{tg}\,\gamma\beta(\bar{r}-1);\ \mathrm{Re} = \frac{\left(\frac{2k}{k+1}RT_1^*\right)^{0.5}(\lambda_r^2+\lambda_\varphi^2)^{0.5}\bar{\Delta}\Delta_0\varepsilon}{2\nu\,(T/T_1^*)^{0.75}}; \tag{2}$$

$$\bar{\Delta} = \Delta/\Delta_0;\ \beta = (r/\Delta)_0;\ \bar{r} = r/r_0.$$

The distribution of dimensionless radial velocity over the diffuser radius is found by solving differential equation (1), while the distribution of the peripheral velocity component is found from Eq. (2).

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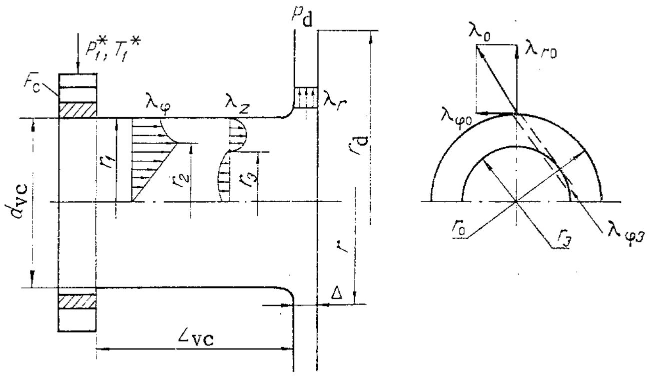
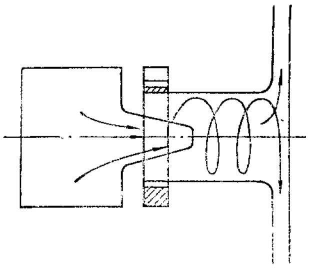


Fig. 1. Diagram of gas flow in a self-evacuating vortex tube.



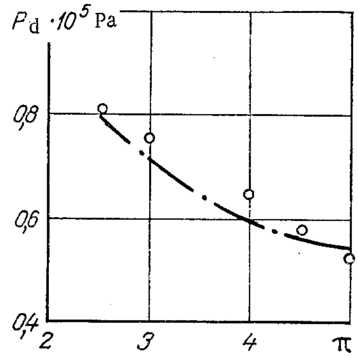


Fig. 2 Fig. 3

Fig. 2. Basic diagram of turbine vacuum pump.

Fig. 3. Dependence of static pressure at the inlet of an SVT diffuser on the available gas expansion; curve obtained by calculation; points obtained by experiment [4].

The separation losses in the diffuser are accounted for with the correction factor χ:

$$\chi = a_0(\beta)\,[\alpha^p - a_1(\beta)]^2 + a_2(\beta), \tag{3}$$

where

$$a_0 = 5{,}29 - 0{,}336\beta + 0{,}885\beta^2\cdot 10^{-2} - 0{,}115\cdot 10^{-3}\beta^3;\quad a_1 = 0{,}28278 + 0{,}0265\beta - 0{,}636\cdot 10^{-3}\beta^2 + 0{,}26167\cdot 10^{-4}\beta^3;$$

$$a_2 = \begin{cases} 1{,}2, & \beta < 10{,}66; \\ 1{,}55 - 0{,}0406\beta + 0{,}00108\beta^2 - 0{,}101\cdot 10^{-4}\beta^3. \end{cases}$$

The pressure-gradient distribution is found from the equation of momentum (1):

$$\overline{P} = \left\{ \frac{\lambda_\varphi^2}{\overline{r}\lambda_r} - \frac{d\lambda_r}{d\overline{r}} - 0{,}046\beta\chi\,\frac{\overline{r}\varepsilon\lambda_r\,(\lambda_r^2 + \lambda_\varphi^2)^{0,5}}{\varepsilon_0\lambda_{r0}\mathrm{Re}^{0,2}} \right\} \frac{\lambda_r\varepsilon}{\chi}. \tag{4}$$

The pressure coefficient is calculated from the formula

$$\xi = \frac{2\Delta p}{\rho_0 v_0^2} = \frac{2\int_1^r \overline{P}\,d\overline{r}}{\varepsilon_0(\lambda_{r0}^2 + \lambda_{\varphi 0}^2)}. \tag{5}$$

The flat divergent diffusers normally used in SVT's and TVP's ($\gamma = 0$) are quite large: $\overline{D}_d = D_d/d_{vc}$ = 4-6 tube diameters. The static pressure at the diffuser outlet is usually known, and most often it is equal to the atmospheric pressure P_n. Taking this into consideration and using the continuity equation, we can determine the radial component of velocity at the diffuser outlet

$$\lambda_{r\mathrm{d}} = -a + \sqrt{a^2 + \frac{k+1}{k-1} - \lambda_{\varphi\mathrm{d}}^2}, \quad a = \frac{3.14 r_0 \Delta_0 \sqrt{\frac{2k}{k+1} R T_\mathrm{d}^*}\, P_\mathrm{d}(k+1)}{(k-1) R T_\mathrm{d}^* G_1}. \tag{6}$$

In Eq. (6), the outlet component of velocity in the peripheral direction is determined in a first approximation without allowance for friction losses. Using the equation of conservation of angular momentum, we obtain an expression for calculating peripheral velocity

$$\lambda_{\varphi_\mathrm{d}} = \lambda_{\varphi 1} / [\bar{r}_\mathrm{d}(1 + 2R_\mathrm{d}/d_\mathrm{vc})]. \tag{7}$$

The rate of flow of compressed air through tangential nozzles is calculated from the formula in [6]:

$$\alpha_\mathrm{c} = 0.98, \; G_1 = \frac{\alpha_\mathrm{c}\left(\frac{2k}{k+1}\right)^{\frac{k+1}{2(k-1)}} P_1^* F_\mathrm{c} q(\lambda_1)\left(\frac{k}{R}\right)^{0.5}}{\sqrt{T_1^*}}, \tag{8}$$

$$T_1^* = T_\mathrm{d}^*. \tag{9}$$

The stagnation temperature in the diffuser is taken equal to the temperature at the inlet of the vortex device.

The assumption that the walls of the SVT, TVP, and diffuser are adiabatic is valid due to the fact that the gas temperature at the outlet of the tangential nozzles and the temperature of the ejected gas is close to room temperature.

The iteration method in [4] can be used to determine the radial velocity at the diffuser inlet. Assigning a value for the radial inlet velocity, we can use Eqs. (1)-(5) to determine the radial velocity at the diffuser outlet. The value of outlet radial velocity obtained by solving differential equation (1) is compared with the corresponding value calculated from Eq. (6). The cycle is repeated until the following assigned accuracy is achieved.

$$|\lambda_{r\mathrm{d}i} - \lambda_{r\mathrm{d}}| / \lambda_{r\mathrm{d}} \leqslant \varepsilon_1. \tag{10}$$

The initial radial velocity corresponding to condition (10) is also the sought value $\lambda_{ro} = \lambda_{roi}$. The static and total pressure at the diffuser inlet is determined from the numerical value of the pressure coefficient:

$$P_{\mathrm{d}0} = P_\mathrm{n}\tau(\lambda_0)\left[\tau(\lambda_0) + \xi_\mathrm{d}\frac{k}{k+1}\lambda_0^2\right]^{-1}, \tag{11}$$

$$P_{\mathrm{d}0}^* = P_\mathrm{n}\pi^{-1}(\lambda_0)\left[1 + \xi_\mathrm{d}\frac{k}{k+1}\lambda_0^2\tau^{-1}(\lambda_0)\right]^{-1}. \tag{12}$$

It is apparent from Fig. 3, showing the distribution of static pressure at the inlet of an SVT diffuser, that the agreement between the pressure calculated from Eq. (12) and the experimental value is good.

In the vortex theory, the entire region of peripheral velocities in a vortex chamber is subdivided into a region of forced vortical flow and a zone of potential flow (Fig. 1). In accordance with the hypothesis of vortex interaction [1], turbulent energy transfer occurs in a vortex chamber when the distribution of thermodynamic parameters corresponds to an adiabatic curve.

Following [1], the static pressure in a forced vortical flow and in the potential region is determined thus:

$$P = P_1\left[1 - \frac{k-1}{2} M_1^2 (1/\bar{r}^2 - 1)\right]^{\frac{k}{k-1}}, \tag{13}$$

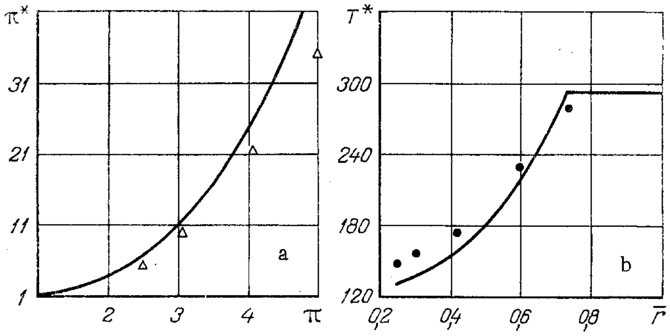


Fig. 4. Dependence of total gas expansion on available gas expansion (a) and radial distribution of total temperature in an SVT (b): a) points are from the data in [4]; $\bar{F}_c = 0.075$; $\Delta_0 = 0.025$; $\bar{D}_d = 4$; $\bar{R}_d = 0.166$; the curve is calculated; b) points are from the data in [3]; the curve shows the calculated results.

$$P = P_1\left[(1/\pi_1)^{\frac{k-1}{k}} + \frac{k-1}{2}M_1^2\frac{\bar{r}^2}{\bar{r}^4}\right]^{\frac{k}{k-1}}. \tag{14}$$

As is known [7], the axial velocity in a vortex chamber changes sign. The radius for which this quantity will be equal to zero is less than the radius of separation of the vortices $\bar{r}_2$. Figure 1 shows the distribution of axial velocity in a vortex chamber. The proposed design method is based on equality of the static and total pressures at the radius of zero axial velocity $\bar{r}_3$ (Fig. 1) and the diffuser inlet (11), (12).

We write the radius $\bar{r}_3$ in the following form:

$$\bar{r}_3 = f\bar{r}_2. \tag{15}$$

The function f is determined from (14):

$$f = \sqrt{\frac{\left(P_{\text{do}}\left(\frac{2}{k+1}\right)^{\frac{k}{1-k}}(P_1^*)^{-1}\right)^{\frac{k-1}{k}} - (1/\pi_1)^{\frac{k-1}{k}}}{0.5(k-1)M_1^2}}\,\bar{r}_2. \tag{16}$$

The radius of separation $\bar{r}_2$, in accordance with [1], is equal to

$$\bar{r}_2^2 = \frac{(k-1)M_1^2}{1-(1/\pi_1)^{\frac{k-1}{k}} + 0.5(k-1)M_1^2}. \tag{17}$$

The assumption made regarding equality of the static pressures at $\bar{r}_3$ and the diffuser inlet can be explained thus: Static pressure is higher at a vortex-chamber radius greater than $\bar{r}_3$ than at the diffuser inlet — axial velocity is directed toward the diffuser at a vortex-chamber radius smaller than $\bar{r}_3$, static pressure is less than the corresponding value at the diffuser inlet — axial velocity is directed away from the diffuser. The equality of the axial velocity at the radius $\bar{r}_3$ to zero corresponds to equality of the static pressures at $\bar{r}_3$ and the diffuser inlet.

The equality of the total pressures at $\bar{r}_3$ and the diffuser inlet can be interpreted, by means of the theory of elemental jets, as gas flows from the radius with zero axial velocity $\bar{r}_3$ to r_0 (inlet radius of diffuser) without losses (to the right in Fig. 1).

The algorithm of the proposed method is as follows:

1) The geometric (F_c, Δ_0, β, D_d, d_{vc}) and regime parameters (π, P_1^*, T_1^*) of the vortex unit are assigned;

2) the static and total pressure at the diffuser inlet are determined from Eqs. (1)-(12);

3) assigning numerical values for the gas expansion, f and the peripheral velocity at the given radius are calculated from Eqs. (15)-(17);

4) the calculated total pressure at r_3 is compared with the total pressure at the diffuser inlet P^*_{d0} (12).

The numerical value of expansion π_1 corresponding to equality of the total pressures at the diffuser inlet and the radius $\bar{r}_3$ is also the solution of the problem.

Figure 4a shows a comparison of experimental data from [3] and calculated values of total expansion π^* in an SVT in relation to the available gas expansion π. It is apparent from the figure that the theoretical and empirical data agree satisfactorily.

The gas-temperature distribution over the vortex-chamber radius, according to [1], is calculated thus:

$$T = T_1 \left[(1/\pi_1)^{\frac{k-1}{k}} + 0.5(k-1) M_1^2 \frac{\bar{r}^2}{\bar{r}^4} \right]; \quad T^* = T\tau^{-1}(\lambda_\varphi).$$

It is apparent from Fig. 4b that the spread of empirical and theoretical distributions over the radius of an SVT vortex chamber reaches 10%.

NOTATION

d_{vc}, diameter of vortex chamber, m; T^*_d, T^*_1, stagnation temperature at the diffuser inlet and vortex-tube inlet, °K; F_c, area of tangential nozzle inlet, m²; R, universal gas constant, J/kg•deg; k, adiabatic curve constant; ε, gas-dynamic density function; $\beta = r_0/\Delta_0$, relative width of diffuser channel at the inlet; $\bar{r} = r/r_0$, relative radius; ν, kinematic viscosity of gas, m²/sec; γ, slope of walls of divergent diffuser, deg; R_d, radius of contact of diffuser and vortex tube, m; $\bar{\Delta} = \Delta/\Delta_0$, relative diffuser gap; $\lambda_r = v_r/\left(\frac{2k}{k+1} RT^*_d\right)^{0.5}$ relative radial velocity; $\lambda_\varphi = v_\varphi/\left(\frac{2k}{k+1} RT^*_d\right)^{0.5}$,— relative peripheral velocity; $\lambda_z = v_z/\left(\frac{2k}{k+1} RT^*_d\right)^{0.5}$, relative axial velocity; $M_1 = v_1/\left(\frac{2k}{k+1} RT_d\right)^{0.5}$, velocity at vortex-tube inlet; P^*_1, pressure at vortex-tube inlet, Pa; P_1, static pressure on the wall of the vortex chamber; P_{ax}, P_n, static pressure on the axis of the vortex device and barometric pressure, Pa; $\pi = P^*_1/P_n$, available gas expansion; $\pi^* = P^*_1/P_{ax}$, total gas expansion; $\pi_1 = P_1/P_{ax}$, gas expansion; $\bar{P} = \frac{dP}{dr} \cdot \frac{r_0}{\rho^*\left(\frac{2k}{k+1} RT^*_d\right)}$ d— dimensionless pressure gradient. Indices: 0, parameters at the diffuser inlet; 1, at the vortex-tube inlet; * and d, stagnation parameters and parameters at the diffuser outlet, respectively.

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SOLUTION OF A TWO-DIMENSIONAL CONJUGATE PROBLEM OF STABILIZED HEAT TRANSFER IN THE LAMINAR FLOW OF A LIQUID IN A CHANNEL

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A two-dimensional problem of conjugate heat exchange is solved by the method of integral boundary-value equations. Heat exchange in a body with cylindrical channels is studied.

Theoretical investigations of convective heat exchange between a solid and a liquid are generally conducted by assigning third-order boundary conditions on the solid—liquid interface. These conditions include the heat-transfer coefficient α, determined a priori. Such a formulation of the problem does not consider the mutual thermal effect of the body and liquid, and heat exchange is independent of the properties of the body or its thermophysical characteristics, dimensions, etc. Thus, it is necessary to examine a so-called conjugate problem, i.e., to simultaneously solve the equations of heat conduction in the body and liquid under the condition of equality of the temperatures and heat fluxes at the interface [1, 2].

One of the approaches to solving conjugate problems is based on the method of integral boundary-value equations [3].

Let the flat wall of the heat exchanger receive a heat flow of intensity q. The heat is removed by a liquid flowing in cylindrical channels of the same radius lying in a plane parallel to the wall. We will assume that the motion of the liquid is laminar and that the heat exchange between the liquid and solid is steady.

Using these assumptions and symmetry conditions, let us formulate the problem of determining the temperature field in the following manner:

In a two-dimensional region D (Fig. 1), consisting of two subregions D_1 (solid) and D_2 (liquid), it is necessary to find the solution to the system of equations

$$\frac{\partial^2 T_1}{\partial x^2}+\frac{\partial^2 T_1}{\partial y^2}=0 \quad (x,\ y)\in D_1, \tag{1}$$

$$c_p\rho W\frac{\partial T_2}{\partial z}=\lambda_2\left(\frac{\partial^2 T_2}{\partial x^2}+\frac{\partial^2 T_2}{\partial y^2}\right) \quad (x,\ y)\in D_2 \tag{2}$$

with the following boundary conditions:

$$\lambda_1\frac{\partial T_1}{\partial n}=q \quad (x,\ y)\in CD,\ T_1=T_2,\ \lambda_1\frac{\partial T_1}{\partial n}=\lambda_2\frac{\partial T_2}{\partial n} \quad (x,\ y)\in AB.$$

We have assigned $\partial T/\partial n = 0$ on the rest of the boundary. Since only the heat flux is assigned on the boundary, the temperature is determined to within a constant, chosen so that the temperature integral over the interface AB is equal to zero.

On the section of thermal stabilization, the derivative $\partial T_2/\partial z$ will be constant. From the condition of heat balance, we find $c_p\rho\frac{\partial T_2}{\partial z}=\frac{2qa}{\pi R^2\overline{W}}$.

The velocity field for laminar flow in a channel of circular cross section is given by Poiseuille's formula [1]:

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